# Fuzzy-Based Stock Selection System through Suitability Index and Position Sizing


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***Abstract*—With the rapid development of quantitative trading, stock selection is an enduring task, that requires consideration of the characteristic of stocks and investment strategies. Fuzzy-set theory is outstanding in modeling abstract characteristics, which is suitable for building the connection between stocks and strategies. In this paper, we propose a stock selection system (SSS) based on the suitability index (SI) and fuzzy-set theory. SSS utilizes position sizing to extract stock characteristics, SI, which expresses not only the characteristics of stocks but also the suitability for strategies. The SI is then transformed by the designed fuzzy-set modules for stock selection and investment. Several stock selection mechanisms are proposed, denoted as SSS-T1, SSS-T2, SSS-WT1, and SSS-WT2. Experimental results show that SSS has a slight improvement in precision, but leads to a significant improvement in investment. The proposed SSS-WT1 and SSS-WT2 systems achieve the highest annual returns of 6.2% and the highest Sharpe ratios of 0.924, outperforming the benchmark with the annual return of 1.9% and the Sharpe ratio of 0.201.**



***Index Terms*—Stock selection, position sizing, suitability index, momentum, contrarian**


## I. Introduction

With the rapid progress of information technology, researchers have shown a keen interest in quantitative trading. Quantitative trading is also known as algorithmic trading, which uses quantitative information to support investment decisions [1]. Data mining is an effective technique for extracting valuable and profitable rules from historical data. Syu et al. [2] proposed a self-managed portfolio system that identifies profitable features through adaptive association mining. To achieve highly complex analysis and prediction, machine learning techniques are widely used in finance. Wu et al. [3] proposed portfolio management systems in the context of deep reinforcement learning. The literature shows the ability of artificial intelligence to support financial investment and decision making.

However, as numerous investment strategies are proposed, stock selection is an enduring task that requires consideration of the characteristics of stocks and strategies. Momentum and contrarian are two well-known contrary characteristics of strategies, but the lack of connection to stocks causes obstacles for stock selection. Fuzzy-set theory is excellent for modeling abstract characteristics in an imprecise environment, and is suitable for establishing links between stocks and strategies. Wu et al. [4] proposed a fuzzy system to quantify the characteristics of stocks through random trading, and achieved high predictive accuracy and 1.5 times profitability over the benchmark. However, due to the high volatility of the financial market, random trading algorithms may produce unstable results in different economic environments. Therefore, stock selection systems should be updated regularly to obtain the latest information. In addition, the papers focused only on the characteristics of stocks, but showed a weak link to investment strategies.

To address these problems, in this paper, we propose a stock selection system (SSS) based on suitability index (SI) and fuzzy-set theory. We first design position sizing algorithms to extract the characteristics of stocks, which are deterministic algorithms with high explainability and no randomness. The extracted characteristics express not only the momentum and contrarian characteristics but also the suitability for the investment strategy, denoted as SI. Subsequently, the SI is transformed by the designed fuzzy-set modules to act as a decision basis for stock selection and further investments. Several stock selection mechanisms are proposed, and SSS with different mechanisms are denoted as SSS-T1, SSS-T2, SSS-WT1, and SSS-WT2. Furthermore, a rolling window mechanism is used to regularly update system parameters, which increases the reliability of real-world investments.

Experimental results show that the proposed systems only have slight improvements in precision, but lead to significant improvements in investment, since the membership value can further assist portfolio management. The proposed WT1 and WT2 systems achieve the highest annual returns of 4.7% and

6.2% and the highest Sharpe ratios of 0.924 and 0.698 (on **TRB** and **GAP** strategies), outperforming the benchmark with the annual returns of 1.0% and 1.9% and the Sharpe ratios of 0.115 and 0.201. In summary, the proposed SSS shows a slight improvement in selection precision and excellent improvements in investment performance and portfolio management, especially in profitability.

## II. Literature Review

In this section, the fuzzy-set theory and the membership functions are first introduced in Section II-A. The characteristics of trading strategies and stocks are then studied in Section II-B. Additionally, the financial indicators to evaluate investment performance are introduced in Section II-C.

### A. Fuzzy-set Theory and Membership Functions

Fuzzy-set is a theory to figuratively grade a concept to a membership degree, and is distinct from probability, because the concept may be multivariate and lack sharply defined criteria [5]. The fuzzy-set has a superior ability to model the uncertainty in the imprecise environment, and provides information for assisting human decision-making.

Most ordinary fuzzy theories belong to type 1 fuzzy-set. Based on the formulation of [6], a type-1 fuzzy-set, $F_1$, maps a set of elements, $X$, to a set of type-1 membership values, $I = [0, 1]$, expressed as:

$$F_1 : \; X \rightarrow I, \tag{1}$$

$$F_1 = \{(x, M_{F_1}(x)) \mid x \in X, \; M_{F_1}(x) \in I\}, \tag{2}$$

where $M_{F_1}$ is the membership function of $F_1$.

However, uncertainty may be contained in the type-1 fuzzy-set; therefore, a higher-order type-2 fuzzy-set [7] was subsequently proposed, which is able to describe uncertainty and capture more information. Based on the formulation of [6], a type-2 fuzzy-set, $F_2$, maps a set of independent variables, $X$, to a set of type-2 membership values, $I^2 = [0, 1] \rightarrow [0, 1]$ (ranges of type-1 membership degree), and can be expressed as:

$$F_2 : \; X \rightarrow I^2, \tag{3}$$

$$F_2 = \{(x, M_{F_2}(x)) \mid x \in X, \; M_{F_2}(x) \in I^2\}, \tag{4}$$

where $M_{F_2}$ is the membership function of $F_2$.

The above-mentioned membership functions map the independent variables to the degrees of truth, ranging between 0 and 1. Among various membership functions, R-, L-, and sigmoid (S-) functions are commonly used [8].

The R- and L- functions are two symmetric Z-shaped functions with two boundaries $(a, b, a < b)$. They have a linear transformation inside the boundary and map values outside the boundary to 0 or 1, which is defined as follows:

$$\mathbf{R}(x) = \begin{cases} 1 & x < a \\ \frac{b-x}{b-a} & a \leq x \leq b \\ 0 & x > b \end{cases}, \tag{5}$$

$$\mathbf{L}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & x > b \end{cases}. \tag{6}$$

Note that the derivatives of the R- and L-functions are not continuous. To avoid the problem of discontinuity, the S-function (particularly, the logistic function) can be used, defined as follows:

$$\mathbf{S}(x) = \frac{1}{1 + e^{-x}} = \frac{1}{1 + a^{-(x+b)}}, \tag{7}$$

where $a$ and $b$ are parameters to control the slope and offset.

### B. Characteristics of Stocks and Trading Strategies

Momentum and contrarian are two well-known categories of trading strategies that describe trading behavior [9], and are contrary concepts that often generate opposite trading decisions. Momentum strategies assume that the movements of stock price are driven by momentum, and a similar price trend will occur in the near future [10]. For example, opening range breakout uses a pre-determined threshold to capture price movements and make investment decisions. Literatures have shown that the opening range breakout strategy is profitable in various markets [11], and is further enhanced by multi-objective optimization [12] and evolutionary algorithms [13]. Contrarian strategies are extended from the mean-reversion phenomenon of stock prices [14], which finds that the price tends to move toward the mean. A prevailing contrarian strategy is the Bollinger bands [15], which utilizes price volatility to detect the signal of mean-reversion.

After a stock position is established, the task of position sizing [16] also involves the concepts of momentum and contrarian. The momentum-type position sizing will increase the position when it earns profits, and will shrink the position when it gets losses, because it believes that the profit and loss will increase with the momentum. On the other hand, the contrarian-type position sizing will shrink the position when it earns profits, and will increase the position when it gets losses, because it believes that the profit and loss will start to reverse as contrarian.

Fixed Ratio (FR) is an intuitive position sizing mechanism [17] that states the relationship between the number of contracts (position size) and the amount of profit should be controlled by a positive fixed ratio, $\delta$. When the momentum-type fixed ratio mechanism is activated, the base price ($BP$) is set to the current price. Once the stock price reaches $\geq BP \times (1 + n \times \delta)$, the $n^{\text{th}}$ grid price, the position should be increased to $n$ units of funds, where $n$ is a positive integer. In other words, when the price increases by each $\delta$ of $BP$, the position should be increased by one new unit of funds. In the fixed ratio contrarian mechanism, once the stock price reaches $\geq BP \times (1 + n \times \delta)$, the position should be reduced by $n$ units of funds, the $n^{\text{th}}$ lattice price. In other words, when the price increases by each $\delta$ of $BP$, one unit of funds should be removed from the position.

### C. Financial Indicators

Several indicators are adopted to evaluate the investment performance. A fundamental indicator is the annual return of investment for the profitability measurement [18]. Let the total profit during the investment period be $Profit$ and the investment cost be $Cost$ and the number of investment days be $Days$. The annual return is defined as:

$$\frac{Profit}{Cost \times \frac{Days}{252}}, \tag{8}$$

where $\frac{Days}{252}$ is the number of investment years, and 252 is the average number of trading days in a year ($\approx 365 \times \frac{5}{7}$).

In considering the trade-off between profitability and risk, the Sharpe ratio [19] is an indicator of how much profit can be obtained by taking a unit of risk. Let the annual return be $AnnRet$, the risk-free return rate be $r_f$, and the standard deviation of daily returns be $Std$. The Sharpe ratio is defined as:

$$\frac{AnnRet - r_f}{Std \times \sqrt{252}}, \tag{9}$$

where $\sqrt{252}$ is a multiplier to adjust daily volatility to annualized volatility. $r_f$ is usually replaced by the treasury yield or even ignored (also in this paper).

## III. Proposed SSS: Stock Selection System

In this section, the architecture of the proposed SSS is presented. To increase the practicability of investment, SSS is executed with rolling window. Specifically, SSS trains with the data of the previous month to obtain the SI, and selects and trades stocks for subsequent months.

The architecture of the proposed SSS is shown in Fig. 1. For a strategy, the training data is first implemented in the position sizing algorithms, obtaining the SI of each stock (Section III-A). The SI of each stock is further transformed by the fuzzy-set theory (Section IV) for stock selection. The two membership values are utilized to quantify the suitability and select the stocks (Section IV-B), and SSS finally applies the strategy to the selected stocks.

### A. Position Sizing

The main idea of position sizing is to adjust the position size according to the unrealized profit and loss. We design momentum and contrarian position sizing algorithms based on the fixed ratio [17]. The algorithms require two parameters, $\delta$ and $MaxPos$, to specify the grid size and the maximum units of position, as shown in Section II-B. The algorithms also require the daily opening price, highest price, and closing price of a stock, denoted as OPEN, HIGH, and CLOSE, which are $D$-dimensional vectors with $D$-day prices, where $D$ is the number of days in the month. Finally, the algorithms output the momentum SI (*MSI*) and the contrarian SI (*CSI*) of the stock. Note that the $\delta$ and $MaxPos$ are set to 2% and 10 in the following experiments.

The momentum position sizing is defined in Algorithm 1. Starting from the first day, initialize the base price ($BP$) to the opening price of the $T$-th day (initialized to 1) and set the current grid index ($Index$, an integer) to 0, which records the grids the price has reached and also the stock units bought (line 4). Also, set the last return rate ($LRet$) to 0 and the list of purchase costs ($Cost$) to empty (line 5).

The fixed ratio invites the purchase of new stock units at integer grids, but the stock price is not continuous within two trading days. Therefore, we need to observe the grid of opening prices ($OGrid$, line 7). If the $OGrid > Index$, the algorithm buys new stock units at the opening price (even if it is not an integer grid, it is the closest and tradable price). In summary, once the $OGrid > Index$, the algorithm buys several units of stock at the opening price until the $Index$ reaches the $OGrid$ (lines 9 to 11).

During the trading day, we then observe the grid of the highest price ($HGrid$, line 8). Once the $HGrid > Index$, the algorithm buys several units of stock at each integer grid price, until the $Index$ reaches the $HGrid$ (lines 12 to 14). Note that the prices from $OGrid$ to $HGrid$ are continuous, and the integer grids between them are tradable. In addition, the total position cannot exceed $MaxPos$, and the cost of buying each position is recorded in $Cost$ (lines 11 and 14).

As for the termination strategy, all positions are sold at the closing price as soon as the closing price $< BP \times (1 + (\text{Index} - 1) \times \delta)$, i.e., the price falls by one grid (lines 18 to 20). The fixed ratio strategy can be restarted from the next day. If the position is not terminated until the last day, $D$, the remaining positions are sold at the closing price of the $D$-th day (lines 21 and 23).

The daily return of each day is recorded in DRet. This is the return on selling all positions at the closing price of the day minus the last return, $LRet$ (lines 15 to 17). Until the last day, the momentum SI (*MSI*) is defined as the Sharpe Ratio of the period (line 25), where Mean($\cdot$) and Std($\cdot$) are the mean and standard deviation of the list.

The contrarian position sizing is similar to Algorithm 1 with the following differences. On each day, if $OGrid > Index$, the algorithm sells several units of stocks at the opening price until the $Index$ reaches the $OGrid$ (lines 9 to 11). During the trading day, once the $HGrid > Index$, the algorithm sells several units of stock at each integer grid price until the $Index$ reaches the $HGrid$ (lines 12 to 14). The gain of selling each position is recorded in $Gain$ (lines 12 and 15). Note that a stock has two SI calculated by the momentum position sizing and the contrarian position sizing, namely *MSI* and *CSI*.

### B. Example of Position Sizing

A brief example of the momentum position sizing is shown in Fig. 2, and assumes that the $\delta$ is 10%. Starting from the $T$-th day, the base price ($BP$) is set to the opening price of 50, and the grid index ($Index$) is initialized to 0. The $HGrid$ of the $T$-th day is $\lfloor (\frac{58}{50} - 1)/10\% \rfloor = 1$. Since $HGrid > Index$, a new position is bought at the grid price 1st, 55 dollars, and the $Index$ is updated to 1. The $OGrid$ and $HGrid$ of the $T+1$-th day are $\lfloor (\frac{62}{50} - 1)/10\% \rfloor = 2$ and $\lfloor (\frac{68}{50} - 1)/10\% \rfloor = 3$. Since $OHrid > Index$, a new position is bought at the

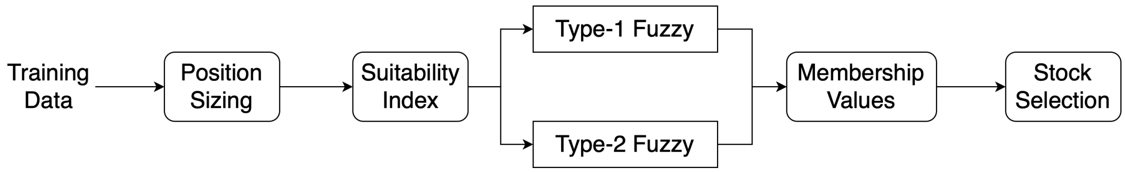



Fig. 1: Architecture of the proposed SSS.

**Algorithm 1** Momentum Position Sizing

**Input:** $\delta$, $MaxPos$, OPEN, HIGH, CLOSE, $D$
**Output:** *MSI*
1: $T$, DRet = 1, []; ▷ day index, daily return
2: **while** $T < D$ **do**
3: **if** True **then**
4: $BP$, $Index$ = OPEN[$T$], 0; ▷ base price, current grid
5: $Cost$, $LRet$ = [], 0; ▷ buying cost, last return
6: **for** $d$ from $T^{\text{st}}$ to $D^{\text{th}}$ day **do**
7: $OGrid$ = $\lfloor$ ( $\frac{\text{OPEN[d]}}{BP}$-1) / $\delta$ $\rfloor$;
8: $HGrid$ = $\lfloor$ ( $\frac{\text{HIGH[d]}}{BP}$-1) / $\delta$ $\rfloor$;
9: **while** $OGrid > Index$ and len($Cost$) $< MaxPoS$ **do**
10: $Inedx$ += 1; ▷ add a position
11: $Cost$ += [ OPEN[d] ];
12: **while** $HGrid > Index$ and len($Cost$) $< MaxPoS$ **do**
13: $Inedx$ += 1; ▷ add a position
14: $Cost$ += [ $BP \cdot$ (1+Index$\cdot\delta$) ];
15: $Ret$ = return rate of selling all stocks at CLOSE[d];
16: DRet += [ $Ret$ - $LRet$ ]; ▷ record the daily return
17: $LRet$ = Ret;
18: **if** CLOSE[d] $< BP \cdot (1 + (\text{Index}-1) \cdot \delta)$ **then**
19: $T$ = $d$;
20: break; ▷ termination condition
21: **if** Not terminated yet **then**
22: $Ret$ = return rate of selling all stocks at CLOSE[$D$];
23: DRet += [ $Ret$ - $LRet$ ];
24: $T$ += 1;
25: *MSI* $= \frac{\text{Mean(DRet)}\sqrt{252}}{\text{Std(DRet)}}$;
26: Return *MSI*.

opening price, 62 dollars, and the $Index$ is updated to 2. As $HGrid > Index$, a new position is bought at the 3$^{\text{rd}}$ grid price, 65 dollars, and the $Index$ is updated to 3.

On $T+2$-th day, the $HGrid$ is $\lfloor(\frac{72}{50}-1)/10\%\rfloor = 4$. Since $HGrid > Index$, a new position is bought at the 4$^{\text{rd}}$ grid price, 70 dollars, and the $Index$ is updated to 4. Even if the lowest price of the $T+2$-th day is lower than the $(Index-1)^{\text{th}} = 3^{\text{nd}}$ grid price, the closing price is still higher than the 3$^{\text{nd}}$ grid price, and the position is not terminated. On $T+3$-th day, the closing price is lower than the grid price $(Index-1)^{\text{th}} = 3^{\text{rd}}$, and all positions are sold at the closing price of $(T+3)$-th day. The same algorithmis applied on the $(T+4)$-th day until the last day, $D$. Note that the contrarian position sizing is similar, and the buying is replaced by selling.

### *C. Suitability Index and Evaluation Strategies*

In this paper, we adopt two well-known strategies, all of which have a momentum- and a contrarian-type perspective (4 strategies in total). The Gap strategy (**GAP**) [20] stipulates that when the opening price is greater than (less than) the last closing price, the signal of momentum GAP, GAP_Mom, (contrarian GAP, GAP_Con) is triggered. Trading range break

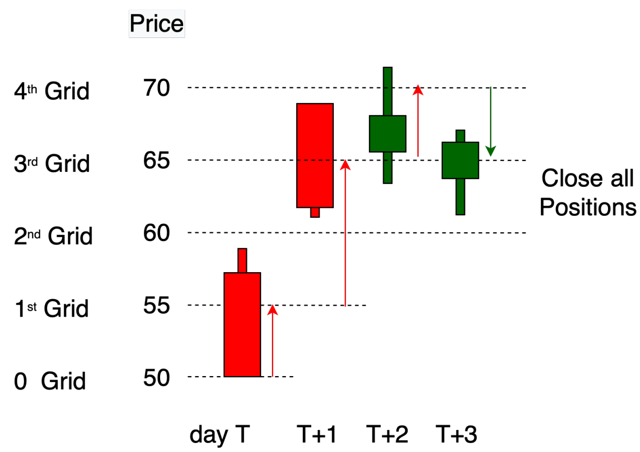



Fig. 2: Schematic diagram of the designed position sizing.

(**TRB**) [21] stipulates that when the opening price is greater than the highest price (less than the lowest price) of the previous $n$ days, the signal of momentum TRB, TRB_Mom, (contrarian TRB, TRB_Con) is triggered. We set the $n$ to 3 in this paper.

The position sizing algorithms defined in Section III-A are also adopted in the above strategies. Specifically, GAP_Mom and TRB_Mom adopt the momentum position sizing of Algorithm 1, with line 3 replaced by the condition of GAP_Mom and TRB_Mom, respectively. The momentum suitability index for GAP_Mom is denoted as *MSI-GAP*, and the momentum suitability index for the TRB_Mom is denoted as *MSI-TRB*. Similarly, GAP_Con and TRB_Con adopt the contrarian position sizing of, replacing line 3 with the condition of GAP_Con and TRB_Con, respectively. The contrarian suitability index for the GAP_Con is denoted as *CSI-GAP*, and the contrarian suitability index for TRB_Con is denoted as *CSI-TRB*.

## IV. Fuzzy-Set Quantifying Modules

In this section, the designed type-1 and type-2 modules are presented. Parameter optimization with rolling window is then presented in Section IV-A, and the stock selection mechanisms are introduced in Section IV-B.

We design the type-1 momentum membership function, T1-MMF, to transform a *MSI* into a type-1 momentum membership value, *T1-MMV*. Similarly, the type-1 contrarian membership function, T1-CMF, is designed to transform a *CSI* to a type-1 contrarian membership value, *T1-CMV*. The *MSI* and *CSI* are in $\mathbb{R}$, and the *T1-MMV* and *T1-CMV* are in

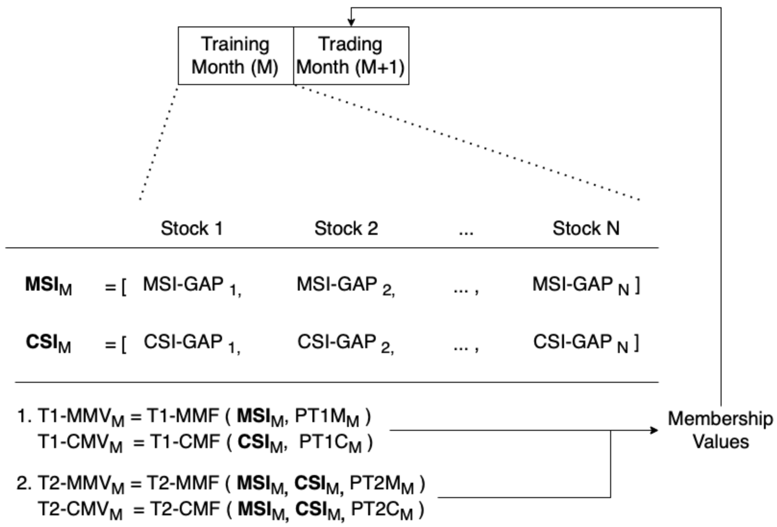



Fig. 3: Designed rolling window and parameter optimization.

$[0, 1]$. The T1-MMF and T1-CMF are designed as S-functions (logistic functions), as shown in Eqs. 10 and 11.

$$\textit{T1-MMV} = \text{T1-MMF}(\textit{MSI}) = \frac{1}{1 + P1^{\ -\textit{MSI}+P2}} \tag{10}$$

$$\textit{T1-CMV} = \text{T1-CMF}(\textit{CSI}) = \frac{1}{1 + P3^{\ -\textit{SCI}+P4}} \tag{11}$$

The $P1$ and $P3$ are parameters to determine the slope, and the $P2$ and $P4$ control the offsets of the functions. Parameters ($P1$ to $P4$) are optimized using the training data of each month, as described in Section IV-A.

Due to the contrary concept of momentum and contrarian characteristics, the linguistic term of momentum (contrarian) can express the inverse linguistic term of contrarian (momentum). Therefore, we arrange the one minus the momentum (contrarian) membership value to represent one side of the type-2 contrarian (momentum) membership value.

The type-2 momentum and contrarian membership functions, T2-MMF and T2-CMF, takes both *MSI* and *CSI* as inputs, and maps to a type-2 momentum membership value, *T2-MMV* (a type-2 contrarian membership value, *T2-CMV*), where *T2-MMV* (*T2-CMV*) is a range of $[0, 1] \rightarrow [0, 1]$. The T2-MMF and T2-CMF are also designed as S-functions, as shown in Eqs. 12 and 13.

$$\begin{aligned} \textit{T2-MMV} &= \text{T2-MMF}(\textit{MSI},\ \textit{CSI}) \\ &= \min(M,\ 1 - C) \rightarrow \max(M,\ 1 - C), \end{aligned} \tag{12}$$

where $M = \frac{1}{1+P1^{\ -\textit{MSI}+P2}}$ and $C = \frac{1}{1+P3^{\ -\textit{CSI}+P4}}$.

$$\begin{aligned} \textit{T2-CMV} &= \text{T2-CMF}(\textit{MSI},\ \textit{CSI}) \\ &= \min(C,\ 1 - M) \rightarrow \max(C,\ 1 - M), \end{aligned} \tag{13}$$

where $M = \frac{1}{1+P1^{\ -\textit{MSI}+P2}}$ and $C = \frac{1}{1+P3^{\ -\textit{CSI}+P4}}$. The *T2-MMV* is a range from the minimum to the maximum of $M$ and $1 - C$, and the *T2-CMV* is a range from the minimum to the maximum of $C$ and $1 - M$. The $M$ and $C$ in Eqs. 12 and 13 are the membership values transformed by T1-MMF and T1-CMF with different parameters ($P1$ to $P4$), which are independently optimized by the training data of each month, as described in Section IV-A.

### A. Parameters Optimization

Parameters in the fuzzy-set modules are independently optimized by the training data of the rolling window. Take the **GAP** as an example, and Fig. 3 illustrates the optimization progresses. In training month $M$, the system first calculates the momentum and contrarian suitability indices of all $N$ stocks, namely $[\textit{MSI-GAP}_1, \cdots, \textit{MSI-GAP}_N]$ and $[\textit{CSI-GAP}_1, \cdots, \textit{CSI-GAP}_N]$, denoted as $\boldsymbol{MSI}_M$ and $\boldsymbol{CSI}_M$, respectively.

For the type-1 fuzzy-set module, the parameters of month $M$ are denoted as $\textit{PT1M}_M$ ($P1$ and $P2$ in T1-MMF, Eq. 10) and $\textit{PT1C}_M$ ($P3$ and $P4$ in T1-CMF, Eq. 11), defined in Eqs. 14.

$$\begin{aligned} \textit{PT1M}_M &= \underset{P1,P2}{\arg\max}\, \text{OBJ}(\textit{T1-MMV}, \boldsymbol{MSI}_M), \\ \textit{PT1C}_M &= \underset{P3,P4}{\arg\max}\, \text{OBJ}(\textit{T1-CMV}, \boldsymbol{CSI}_M), \end{aligned} \tag{14}$$

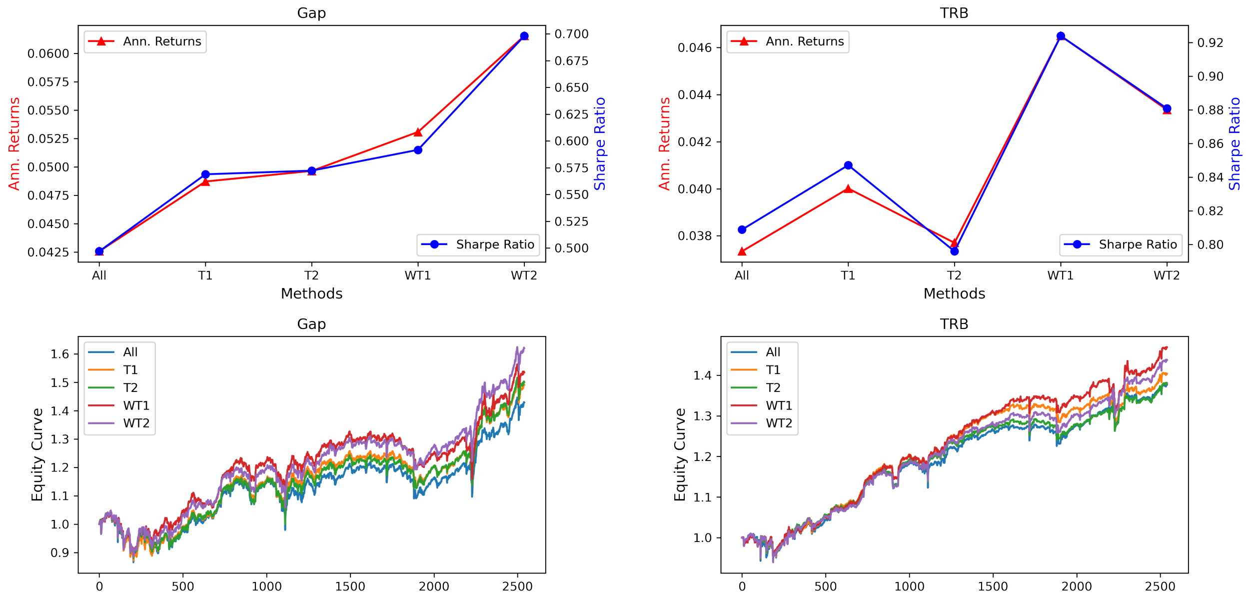
where:

$$\begin{aligned} \textit{T1-MMV} &= \text{T1-MMF}(\boldsymbol{MSI}_{M-1}, [P1, P2]), \\ \textit{T1-CMV} &= \text{T1-CMF}(\boldsymbol{CSI}_{M-1}, [P3, P4]). \end{aligned} \tag{15}$$

The *T1-MMV* states the type-1 momentum membership values transformed from $\boldsymbol{MSI}_{M-1}$ by T1-MMF with parameters $P1$ and $P2$. Similarly, *T1-CMV* states the type-1 contrarian membership values transformed from $\boldsymbol{CSI}_{M-1}$ by T1-CMF with parameters $P3$ and $P4$. OBJ is the objective function, and is defined as:

TABLE I: Performance evaluation of the systems on different strategies

| | ALL | | | SSS-T1 | | | SSS-T2 | | | SSS-WT1 | | | SSS-WT2 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Pre. | Ret. | Sharpe | Pre. | Ret. | Sharpe | Pre. | Ret. | Sharpe | Pre. | Ret. | Sharpe | Pre. | Ret. | Sharpe |
| **GAP** | 41% | 4.3% | 0.497 | 43% | 4.9% | 0.569 | **44%** | 5.0% | 0.572 | 43% | **5.3%** | **0.592** | **44%** | **6.2%** | **0.698** |
| **TRB** | 42% | 3.7% | 0.809 | 42% | 4.0% | 0.847 | **43%** | 3.8% | 0.796 | 42% | **4.7%** | **0.924** | **43%** | **4.3%** | **0.881** |




(a) Gap

(b) 3H3L

Fig. 4: Investment performance and equity curves of the systems on different strategies.

$$\text{OBJ}(MV, SI) = \frac{\text{Mean}(\overline{SI})}{\text{Std}(\overline{SI})}, \tag{16}$$

where:

$$\overline{SI} = [SI_i \mid MV_i > 0.5, \quad i = 1, \cdots, \text{length of } MV]. \tag{17}$$

The OBJ is inspired by the concept of the Sharpe ratio to consider both the average and variance of performance. In summary, we search the parameters in T1-MMF and T1-CMF that transform the SI of the previous month into membership values and have an optimal selected performance for the following months.

For the type-2 fuzzy-set module, the parameters of month $M$ are represented as $PT2M_M$ ($P1$ to $P4$ in T2-MMF, Eq. 12) and $PT2C_M$ ($P1$ to $P4$ in T2-CMF, Eq. 13), defined in Eqs. 18.

$$\begin{aligned} PT2M_M &= \underset{P1,P2,P3,P4}{\arg\max}\ \text{OBJ}(\text{Mid}(\mathit{T2\text{-}MMV}), \boldsymbol{MSI}_M), \\ PT2C_M &= \underset{P1,P2,P3,P4}{\arg\max}\ \text{OBJ}(\text{Mid}(\mathit{T2\text{-}CMV}), \boldsymbol{CSI}_M), \end{aligned} \tag{18}$$

where:

$$\begin{aligned} \mathit{T2\text{-}MMV} &= \text{T2-MMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P1, \cdots, P4]), \\ \mathit{T2\text{-}CMV} &= \text{T2-CMF}(\boldsymbol{MSI}_{M-1}, \boldsymbol{CSI}_{M-1}, [P1, \cdots, P4]). \end{aligned} \tag{19}$$

Eq. 18 is quite similar to Eq. 14. However, since the type-2 membership value is a range, we take the midpoints, Mid($\cdot$), to calculate the objective value in Eq. 18.

Note that the parameters are optimized in discrete spaces. The $P1$ and $P3$ are slopes of S-functions and in a search space of $[1 + 0.2 \times i \mid i = 1, \cdots, 10]$. The $P2$ and $P4$ are offsets of S-functions and in a search space of $[0.5 \times i \mid i = -5, \cdots, 5]$.

### B. Stock Selection Mechanisms

With the optimized parameters, the membership values of each stock can be determined, providing information for stock selection in the following trading month. In this section, four stock selection mechanisms are proposed, including T1, T2, WT1, WT2, and SSS with the stock selection mechanisms are named SSS-T1, SSS-T2, SSS-WT1, and SSS-WT2, respectively. Take the **GAP** as an example. The T1 (T2) mechanism selects stocks with *T1-MMV* (*T2-MMV*) greater than 0.5 for the GAP_Mom strategy, and selects stocks with *T1-CMV* (*T2-CMV*) greater than 0.5 for the GAP_Con strategy.

Furthermore, the size of the membership value can not only select stocks but also assist portfolio management. The WT1 and WT2 mechanisms are the weighted T1 and T2 mechanisms, defined as follows. For the selected stock $i$ with a momentum membership value of $MMV_i$, the system set $w_i$ to $2 \times (\text{MFV}_i - 0.5)$, and the final asset weight invested in stock $i$ is $\frac{w_i}{\sum_i w_i}$. The same mechanism applies to contrarian strategies.

## V. Experimental Results

### A. Data Usage

In this paper, we investigate the stocks in Taiwan stock market and apply them to the proposed SSS. There are 1,230 stocks that exist from January 1, 2011 to at least June 30, 2021. To avoid the impact of stock splits and mergers and capital reductions and increases, we remove stocks with the highest (lowest) daily price is more than 1.1 times (less than 0.9 times) the previous closing price (since upper limit for price increases and decreases in one day is $\pm 10\%$). After data cleaning, 307 stocks remain in the list of stocks.

As for the investment mechanism, for each strategy (**Gap** and **TRB**), we combine the results of the momentum and contrarian type strategies to obtain the neutral and comprehensive evaluation. In addition, we take the benchmark system of ALL [22] for comparison, which is widely recognized in the financial field [23]. The ALL invests in all stocks with equal weight, considering all stocks as both momentum and contrarian. Note that we ignore transaction costs in this paper.

### B. Performance evaluation

In this section, we evaluate the performance of the proposed SSS through descriptive statistics and investment results, listed in Table I. Since the system is designed for stock selection, the correct selection will result in a profit; therefore, we adopt precision as a measurement. For the investment measurements, we adopt the annual return and Sharpe ratio. Fig. 4 presents the investment results and equity curves of the systems.

In Table I, the columns represent the benchmark system and the proposed four stock selection systems, including T1, T2, WT1, and WT2. The rows in the table represent the two investigated strategies, and the bolded values are the top-2 performances of the strategy (row). Note that the only difference between T1 and WT1 (T2 and WT2) is the weighting mechanism, which doesn't affect the selected stocks, and T1 and WT1 (T2 and WT2) have the same precision.

Experimental results show that the T2 and WT2 have the best precision of stock selection, and have only slight improvements over the benchmark system, ALL. However, the membership value can further assist portfolio management. Even a small increase in precision can lead to a significant improvement in investment.

In terms of investment results, the proposed WT1 and WT2 systems are outstanding, with annual returns and Sharpe ratios ranking in the top-2 of all systems. The WT1 system achieves the highest annual return of 4.7% and the highest Sharpe ratio of 0.924 on **TRB**, outperforming ALL with an annual return of 1.0% and a Sharpe ratio of 0.115. The WT2 system achieves the highest annual return of 6.2% and the highest Sharpe ratio of 0.698 on **GAP**, outperforming ALL with an annual return of 1.9% and a Sharpe ratio of 0.201.

The same phenomenon can be found in the equity curves of Fig. 4, where the red and purple curves (WT1 and WT2 systems) gradually separate and lead to others. Furthermore, the proposed systems suppress ALL, the blue curve, in the second half of the period.

## VI. Conclusions

In this paper, we propose a stock selection system (SSS) based on suitability index (SI) and fuzzy-set theory. We first design position sizing algorithms to extract the characteristics of stocks, SI, which express not only the momentum and contrarian characteristics but also the suitability for the strategy. The SI is then transformed by the designed fuzzy-set modules for stock selection and further investments. Several stock selection mechanisms are proposed, denoted as SSS-T1, SSS-T2, SSS-WT1, and SSS-WT2. Furthermore, we adopt a rolling window mechanism to update system parameters periodically. Experimental results show that SSS has a slight improvement in precision, but leads to a significant improvement in investment, since the membership value can further assist portfolio management. The proposed WT1 and WT2 systems achieve the highest annual returns of 6.2% and the highest Sharpe ratios of 0.924, outperforming ALL with the annual return of 1.9% and the Sharpe ratio of 0.201. In the future, we will examine the uncertainty of the selected stocks and develop other investment strategies to possibly get a better performance on the investment.

## Acknowledgments

The work was supported in part by the Ministry of Science and Technology of Taiwan, under contract MOST110-2222-E-182-003-.

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